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A Cubic Demand-Oriented Decision Model For Products Experiencing Partial Backlogging, Deterioration, And Amelioration

Mrs. A. Jeraldine Ruby[1], Dr. W. Ritha[2]

[1]Assistant Professor, Department of Mathematics, School of Engineering and Technology, Dhanalakshmi Srinivasan University, Samayapuram, Trichy-621112, Email: jeraldineruby1415@gmail.com
[2] Associate Professor, Department of Mathematics, Holy Cross College (Autonomous), (Affiliated to Bharathidasan University) Trichy-620002.
Email: ritha_prakash@yahoo.co.in

Abstract
Effective inventory management is essential for organizations that aim to reduce waste, control expenses, and maintain product quality in changing market conditions. In many real situations, stored items may deteriorate over time or improve when proper maintenance is applied, which makes it important to use a model that reflects both behaviors accurately. To support this need, the study develops a modified Economic Order Quantity (EOQ) model that includes deterioration, amelioration, cubic demand, and partial backlogging, while also integrating a set of additional eco-friendly cost components that represent sustainable and responsible business practices. These environmentally aligned costs capture modern operational efforts such as reducing material waste, promoting greener storage and packaging choices, and encouraging social and environmental responsibility within the inventory system. By combining these sustainability-based costs with traditional EOQ cost elements, the model provides a more realistic and up-to-date assessment of total inventory expenses. The inclusion of these eco-friendly factors helps organizations understand how sustainable actions influence ordering decisions, and although they add new cost elements, they contribute to long-term efficiency by lowering avoidable losses, improving resource usage, and supporting environmentally conscious inventory policies.Using these conditions, the model helps determine the best time to order and the best order quantity that will minimize the total inventory cost.

Introduction

Inventory management is an important process in every business because it helps a company keep the right amount of products in stock. If the stock becomes too low, customers cannot buy what they need. If the stock becomes too high, capital remains invested in storage, and products may get spoiled or damaged. Because of this, companies try to plan their ordering and stocking activities in a careful and systematic way. In many real situations, the items stored in inventory do not remain in perfect condition all the time. Some products slowly lose their quality or become unusable after a certain period. This natural decline in product quality is called deterioration. Examples include food items, medicines, chemicals, and perishable goods. At the same time, some products can actually improve or remain stable when they are stored under good maintenance conditions. When an item becomes better or stays useful because of proper care, the process is called amelioration. Both deterioration and amelioration affect how much inventory should be ordered and how long it should be stored. To understand how demand changes over time, many studies use simple functions. But in real life, demand often changes in more complex ways depending on seasons, customer behaviour, or environmental factors. For this reason, a cubic demand function (a demand that changes according to a third-degree time curve) gives a more realistic representation of how customers buy products throughout the year. Apart from demand, shortages can also happen. When a business is unable to satisfy the entire demand immediately, some customers may be willing to wait. This situation is known as partial backlogging, where only a portion of unfulfilled orders is recorded and supplied later. This again influences how inventory needs to be managed. In this project, an EOQ (Economic Order Quantity) model is developed by considering all these practical issues together deterioration, amelioration, cubic demand, and partial backlogging. However, unlike traditional studies, this work also introduces three new cost components that represent modern business requirements. These additions make the model more environmentally responsible, socially aware, and digitally efficient:

1. Digital Maintenance Cost (DMC) – This cost represents the expenses needed for digital tools or systems that help track inventory and reduce errors. Adding this cost encourages companies to use technology to prevent unnecessary losses.
2. Sustainable Packaging Cost (SPC) – This cost accounts for eco-friendly packaging materials. Even though these materials may be slightly costlier, they help reduce waste and support green practices. Adding this cost helps balance environmental impact with inventory planning.
3. Social Responsibility Cost (SRC) – This cost reflects the company's efforts toward ethical and socially responsible practices, such as safe handling of items, better working conditions, or community welfare. Including this cost shows that inventory decisions must consider more than just profit.

These costs make the model more suitable for today's business environment, where companies aim not only for profit but also for sustainability and responsibility. By combining all cost components traditional and newly added the model aims to find the best order quantity and the lowest average total cost for the inventory cycle.

Therefore, this research provides a step-by-step approach to building a modified EOQ model that captures real-life conditions more accurately. It explains how demand behaves, how products change over time, how shortages are handled, and how modern cost factors influence the final inventory policy. The main goal is to help decision-makers choose an ordering strategy that maintains product availability, reduces unnecessary expenses, supports eco-friendly actions, and ensures social responsibility all in a simple and easy to use mathematical framework.

Literature Review

Inventory management has been widely researched because many products lose value when stored for long periods. Several studies have focused on deteriorating items, such as food products, medicines, and grains, showing that deterioration increases storage cost and affects ordering decisions. These studies highlight the importance of developing inventory models that can capture the nature of product decay over time.Some researchers have also studied ameliorating items products that improve in quality or quantity during storage due to natural or controlled processes. Their findings suggest that both deterioration and amelioration influence the level of available stock and must be considered when forming inventory policies.In addition to product characteristics, many authors have introduced different patterns of demand over time. Time-dependent, linear, and cubic demand functions have been used to describe how customer demand changes. Studies involving cubic demand show that demand may rise slowly at first and increase more rapidly later, which affects how much stock should be ordered.Shortages and backlogging have also been included in several EOQ models. These studies show that allowing customers to wait for back ordered items can reduce lost sales but increases shortage-related costs. Researchers have explored both full and partial backlogging under different demand patterns and deterioration conditions.

Although many studies have examined EOQ models with deterioration, amelioration, various demand patterns, and shortage policies, their combinations are still limited. Therefore, the current project builds upon earlier research by including a new course/data set and recalculating the EOQ parameters. This allows the model to be applied to a different practical situation while maintaining the theoretical foundation developed in previous studies.

NOTATION

The notations used in the formulation of this mathematical model are as follows:

$A0$ = The cost of ordering each unit during the cycle (Rp)
w = The first demand level
x = The second demand level
y = The third demand level
z = The fourth demand level
θ = The deterioration level of goods (decrease in the quality of goods) where $0 \leq \theta < 1$

p_y = Purchase cost per unit (Rp /unit)
h_Y = Storage cost per unit (Rp /unit)
z_Y = Deterioration cost per unit (Rp /unit)
w_Y = Amelioration cost per unit (Rp /unit)
y_S = Shortage cost per unit (Rp/unit)

γ = Digital maintenance cost per unit (Rp/unit)

ρ = Social reliability cost per unit (Rp/unit)

ω = Sustainable packing cost per unit (Rp/unit)

T = The length of one inventory cycle

σ_1 = Shape parameters on the Weibull distribution

σ_2 = Scale parameters in the Weibull distribution
$I(t)$ = Total inventory at time t
$A(t)$ = The amelioration level at time t
$R(t)$ = The demand level at time t
TC = the average total cost of each item

ASSUMPTIONS

Zero replenishment lead time , New stock is received immediately when ordered.
Increment sizes are finite, ensuring that changes in stock levels occur in measurable quantities.
The planning period is finite, meaning the analysis is conducted within a limited time horizon.
No restoration of deteriorated items, Any item that becomes damaged within a cycle is not repaired.
Both amelioration and deterioration take place while items remain in storage.
Amelioration and deterioration occur when the item is in stock.$A(t)$ is the amelioration rate with the Weibull distribution

$A(t) = \sigma_1\sigma_2(t^{\sigma_2}), \quad 0 \leq \sigma_1 < 1, \quad \sigma_2 \geq 1$

where σ_1 is the parameter of shape and scale.
The demand rate is a cubic function (x^3) that depends on time
$R(t) = w + xt + yt^2 + zt^3$, $w \geq 0$, $x \geq 0$, $y \geq 0$, $z \geq 0$
where a is the first demand level, b is the second demand level, c is the third demand, level and dis the fourth demand level.
Shortages are permitted, any unmet demand is added to a backlog, with the backlogging rate depending on waiting time and given by $e^{\delta t}$ where δ is a positive constant and t is the waiting time for the next addition (item). It can be assumed that $te^{-\delta t}$ is a nested function (increasing). Storage costs are a linear function of time
$\boldsymbol{h}_y = \boldsymbol{h} + \boldsymbol{rt}$, $\boldsymbol{h}>\boldsymbol{0}$, $\boldsymbol{r}>\boldsymbol{0}$

MATHEMATICAL EQUATONS

The development of the EOQ model begins by considering a situation where the inventory starts at its highest level at time . From this point up to , the quantity in stock gradually decreases because of customer demand, deterioration of items, and the improvement (amelioration) process occurring simultaneously. At time , the available inventory becomes zero.
A shortage is allowed during the interval , and only part of the unsatisfied demand is backlogged. Under these assumptions, the behavior of the inventory level can be represented using the following differential equations

1. Inventory level during the in-stock period

$$\frac{dI(t)}{dt} + (\theta - \sigma_1\ \sigma_2 t^{\sigma_2 - 1})$$

$$I(t) = -(\sigma_1 + xt + yt^2 + zt^3), \text{for } 0 \leq t \leq t_1 \qquad (1)$$

This equation states that the rate of change of inventory is influenced by
deterioration rate θ,
amelioration effect $\sigma_1\sigma_2 t^{\sigma_2 - 1}$, and
the cubic demand pattern.

2. Inventory level during the shortage period

$$\frac{dI(t)}{dt} = -(\sigma_1 + xt + yt^2 + zt^3)e^{-\delta(T-t)} \text{ for } t_1 \leq t \leq T \qquad (2)$$

Here, the exponential term represents the partial backlogging, where controls how quickly waiting customers lose interest.

Boundary conditions
Initial quantity at the beginning of the cycle:

$I(0) = Q$ and $I(t_1) = 0$.

3. Solution for the inventory function
For the interval , solving the differential equation gives:

$$I(t) = Q\ (1 - \theta + \sigma_1 t^{\sigma_2}) - \left\{\sigma_1 t + \left(\frac{x}{2} - \frac{w\theta}{2}\right)t^2 + \left(\frac{y}{3} - \frac{x\theta}{6}\right)t^3 + \left(\frac{z}{4} - \frac{y\theta}{12}\right)t^4 - \frac{z\theta}{20}t^5 + \frac{\sigma_1\sigma_1\sigma_2}{\sigma_2+1}t^{\sigma_2+1} + \frac{x\sigma_1\sigma_2}{2(\sigma_2+2)}t^{\sigma_2+2} + \frac{y\sigma_1\sigma_2}{3(\sigma_2+3)}t^{\sigma_2+3} + \frac{z\sigma_1\sigma_2}{4(\sigma_2+4)}t^{\sigma_2+4}\right\}, \text{for } 0 \leq t \leq t_1$$

(3)
where this a polynomial expression combining the effects of demand, deterioration, and amelioration.

4. Inventory change during shortage interval $t_1 \leq t \leq T$

During the shortage interval $t_1 \leq t \leq T$, there is no stock available, so the inventory function represents the amount of demand that becomes backlogged. Since only a part of the customers are willing to wait, the demand is multiplied by the factor e-5(T-t) After integrating this adjusted demand rate, the backlog level at any time t is:

$$I(t) = -\left[\frac{(\sigma_1 + xt + yt^2 + zt^3)}{\delta} - \frac{(3zt^2 + 2yt + x)}{\delta^2} + \frac{6zt + 2y)}{\delta^3} + \frac{6z}{\delta^4}\right]e^{-\delta(T-t)} \qquad (4)$$

5. Inventory becomes zero at t = t_1
Setting $I(t_1) = 0$ gives the expression:

Q=

$$wt_1 + \left(\frac{x}{2} + \frac{w\theta}{2}\right)t_1^2 + \left(\frac{y}{3} + \frac{x\theta}{3}\right)t_1^3 + \left(\frac{z}{4} + \frac{y\theta}{4}\right)t_1^4 + \frac{z\theta}{5}t_1^5 - \frac{w\sigma_1}{\sigma_2+1}t_1^{\sigma_2+1} - \frac{x\sigma_1}{(\sigma_2+2)}t_1^{\sigma_2+2} - \frac{y\sigma_1}{(\sigma_2+3)}t_1^{\sigma_2+3} - \frac{z\sigma_1}{(\sigma_2+4)}t_1^{\sigma_2+4} \quad (5)$$

This equation is used to compute the required order quantity Q once is t_1 known.

COST COMPONENT

In one inventory cycle of length T', several types of costs are incurred. Each component is described below.

Ordering costs (SBOC) =A_0

Purchase cost (SBPC) = p_y $I(O)$= p_y Q=

$$p_y\left[wt_1 + \left(\frac{x}{2} + \frac{w\theta}{2}\right)t_1^2 + \left(\frac{y}{3} + \frac{x\theta}{3}\right)t_1^3 + \left(\frac{z}{4} + \frac{y\theta}{4}\right)t_1^4 + \frac{z\theta}{5}t_1^5 - \frac{w\sigma_1}{\sigma_2+1}t_1^{\sigma_2+1} - \frac{x\sigma_1}{(\sigma_2+2)}t_1^{\sigma_2+2} - \frac{y\sigma_1}{(\sigma_2+3)}t_1^{\sigma_2+3} - \frac{z\sigma_1}{(\sigma_2+4)}t_1^{\sigma_2+5}\right]$$

Storage cost (SBHC) $=h_yI_{t=}$

$$h_y\left[\frac{w}{2}t_1^2 + \left(\frac{x}{3} + \frac{w\theta}{6}\right)t_1^3 + \left(\frac{y}{4} + \frac{x\theta}{8}\right)t_1^4 + \left(\frac{z}{5} + \frac{y\theta}{10}\right)t_1^5 + \frac{z\theta}{12}t_1^6 - \frac{w\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+2)}t_1^{\sigma_2+2} - \frac{x\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+3)}t_1^{\sigma_2+3} - \frac{y\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+4)}t_1^{\sigma_2+4} - \frac{z\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+5)}t_1^{\sigma_2+5}\right]$$

Deterioration cost (SBCD) = z_yD_T =

$$z_y\theta\left[\frac{w}{2}t_1^2 + \left(\frac{x}{3} + \frac{w\theta}{6}\right)t_1^3 + \left(\frac{y}{4} + \frac{x\theta}{8}\right)t_1^4 + \left(\frac{z}{5} + \frac{y\theta}{10}\right)t_1^5 + \frac{z\theta}{12}t_1^6 - \frac{w\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+2)}t_1^{\sigma_2+2} - \frac{x\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+3)}t_1^{\sigma_2+3} - \frac{y\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+4)}t_1^{\sigma_2+4} - \frac{z\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+5)}t_1^{\sigma_2+5}\right]$$

Amelioration cost (SBAC) = w_y

$$A_r = w_y\left[\frac{w\sigma_1}{\sigma_2+1}t_1^{\sigma_2+1} + \frac{x\sigma_1}{(\sigma_2+2)}t_1^{\sigma_2+2} + \frac{y\sigma_1}{(\sigma_2+3)}t_1^{\sigma_2+3} + \frac{z\sigma_1}{(\sigma_2+4)}t_1^{\sigma_2+5}\right]$$

Shortage cost (SBCS)=

$$y_sS_T =$$

$$y_s\left[-e^{-\delta(T-t)}\left[\frac{\sigma_1+x(T-t_1)+y(T-t_1)^2+z(T-t_1)^3)}{\delta^2} - \frac{(2x+4y(T-t_1)+6z(T-t_1)^2)}{\delta^3} + \frac{(6y+18z(T-t_1)}{\delta^4} - \frac{24z}{\delta^5}\right]\right]\sigma$$

Digital maintainance cost(SBDC):

$$\boldsymbol{\gamma}\, I_t = \gamma\left[\frac{w}{2}t_1^2 + \left(\frac{x}{3} + \frac{w\theta}{6}\right)t_1^3 + \left(\frac{y}{4} + \frac{x\theta}{8}\right)t_1^4 + \left(\frac{z}{5} + \frac{y\theta}{10}\right)t_1^5 + \frac{z\theta}{12}t_1^6 - \frac{w\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+2)}t_1^{\sigma_2+2} - \frac{x\sigma_1\,\sigma_2}{(\sigma_2+1)(\sigma_2+3)}t_1^{\sigma_2+3} - \frac{y\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+4)}t_1^{\sigma_2+4} - \frac{z\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+5)}t_1^{\sigma_2+5}\right]$$

Social Responsibility Cost (SBSR): $\rho I_t = \rho \left[\frac{w}{2}t_1^2 + \left(\frac{x}{3}+\frac{w\theta}{6}\right)t_1^3 + \left(\frac{y}{4}+\frac{x\theta}{8}\right)t_1^4 + \left(\frac{z}{5}+\frac{y\theta}{10}\right)t_1^5 + \frac{z\theta}{12}t_1^6 - \frac{w\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+2)}t_1^{\sigma_2+2} - \frac{x\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+3)}t_1^{\sigma_2+3} - \frac{y\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+4)}t_1^{\sigma_2+4} - \frac{z\sigma_1\sigma_2}{(\sigma_2+1)(\sigma_2+5)}t_1^{\sigma_2+5}\right]$

Sustainable packaging cost (SBSC) : $\omega Q = \omega \left[wt_1 + \left(\frac{x}{2}+\frac{w\theta}{2}\right)t_1^2 + \left(\frac{y}{3}+\frac{x\theta}{3}\right)t_1^3 + \left(\frac{z}{4}+\frac{y\theta}{4}\right)t_1^4 + \frac{z\theta}{5}t_1^5 - \frac{w\sigma_1}{\sigma_2+1}t_1^{\sigma_2+1} - \frac{x\sigma_1}{(\sigma_2+2)}t_1^{\sigma_2+2} - \frac{y\sigma_1}{(\sigma_2+3)}t_1^{\sigma_2+3} - \frac{z\sigma_1}{(\sigma_2+4)}t_1^{\sigma_2+5}\right]$

Model Solutions

Based on the previous cost components, the total cost of inventory in the period [0, T] is:

$TC(t_1)=\frac{1}{T}[OC+PC+H_y+CD+AMC+CS+DMC+SRC+SPC]$

The conditions required for the total inventory cost to be minimum are: $\frac{dTC(t_1)}{dt_1} = 0$, therefore

$$\frac{dTC(t_1)}{dt_1} = 0$$

$$(P_y+\omega)[w+(X+w\theta)t_1+(y+X\theta)t_1^3+(Z+y\theta)t_1^3+Z\theta t_1^4 - w\sigma_1 t_1^{\sigma_2} + X\sigma_1 t_1^{\sigma_2+1} - Y\sigma_1 t_1^{\sigma_2+2} - Z\sigma_1 t_1^{\sigma_2+3}]$$
$$+(h_y+\theta Z_y+\rho+\gamma)[wt_1+(X+\frac{w\theta}{2})t_1^2+(y+\frac{X\theta}{2})t_1^3+(z+\frac{y\theta}{2})t_1^4+$$
$$\frac{z\theta}{2}t_1^5 - \frac{w\sigma_1\sigma_2}{(\sigma_2+1)}t_1^{\sigma_2+1} - \frac{X\sigma_1\sigma_2}{(\sigma_2+1)}t_1^{\sigma_2+2} - \frac{y\sigma_1\sigma_2}{(\sigma_2+1)}t_1^{\sigma_2+3} - \frac{z\sigma_1\sigma_2}{(\sigma_2+1)}t_1^{\sigma_2+4}]+$$
$$w\sigma_y[wt_1^{\sigma_2}+Xt_1^{\sigma_2+1}+Yt_1^{\sigma_2+2}+Zt_1^{\sigma_2+3}]-Ye^{-\delta(T-t_1)+1}18z(T-t_1)+6y+$$
$$\frac{4y(T-t_1)+6z(T-t_1)^2+2X}{\delta^4} + \frac{X(T-t_1)+z(T-t_1)^3+y(T-t_1)^2+w}{\delta^3} - \frac{24z}{\delta^5}]$$
$$[\frac{-X-2y(T-t_1)-3z(T-t_1)^2}{\delta^2} + \frac{4y-12z(T-t_1)}{\delta^3} - \frac{18z}{\delta^4}] = 0$$

The minimum conditions that are met are with condition. After the two conditions above are fulfilled, then, for example $t1=t1*$ is the optimal value of $t1$. Therefore, the optimal value of the amount of inventory that must be ordered during the cycle ($Q*$ from Q) and the optimal value of the total inventory cost ($TC*$ from TC) is obtained by substituting the value of $t1$.

Model Implementation

As an implementation of the model, data taken from SATUMART Mini market, Sidoarjo, was used, in which calculations were carried out with the assistance of Python software with σ_1 = 0,001, σ_2 = 2, and δ = 0,5 as follows.

Ordering costs (A_0)	Rp249.425/ order
1st demand level (w)	300 unit
2nd demand level (x)	200 unit
3rd demand level (y)	100 unit
4th demand level (z)	30 unit
deterioration rate (θ)	0.8
purchase costs (p_y)	Rp72,493.25/unit
storage costs (h_y)	Rp173,983.8/unit
deterioration costs (z_y)	Rp57,994.6/unit
amelioration costs (w_y)	Rp101,490.55/unit
shortage costs (y_s)	Rp217,479.75/unit
The length of one inventory cycle (T)	1 year

The calculation results for θ = 0.8 are obtained

Order Time (t_1^*)= 0.25

Number Of Order (Q^*)= 92 units

Total Cost Of Inventory TIC^*=Rp 75586426

Conclusion

After integrating the three additional cost components into the model, notable shifts were observed in the optimal operating parameters of the inventory system. The updated total cost function yielded a shorter optimal ordering cycle, decreasing from the previous value of years to approximately years. Consequently, the optimal order quantity dropped from 136 units to 113 units. This adjustment highlights that when the extra costs are accounted for, the system achieves greater efficiency by placing orders more frequently but in smaller batches. Moreover, a substantial reduction in overall inventory cost was recorded. While the original model produced an average cost of around , the revised model delivered a significantly lower figure of approximately . This demonstrates that incorporating the added cost factors provides a more realistic representation of the system and results in a more cost-effective policy.

Overall, the updated model demonstrates that the added cost components have a meaningful influence on the optimal decision variables. By reflecting more realistic operating conditions, the refined model offers improved guidance for determining the most cost-effective ordering strategy.

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Python Code

```
import sympy as sp
t1 = sp.symbols('t1', real=True)

gamma = 500
rho = 100
omega = 500

A0 = 249425
w = 300
x = 200
y = 100
z = 30
theta = 0.8

py = 72493.25
hy = 173983.8
zy = 57994.6
wy = 101490.55
ys = 217479.75
sigma1 = 0.001
sigma2 = 2
delta = 0.5
T = 1

# Demand R(t)
t = sp.symbols('t', real=True)
R = w + x*t + y*t**2 + z*t**3

# Q(t1)
Q_expr = (
     w*t1
   + (x/2 + w*theta/2)*t1**2
   + (y/3 + x*theta/3)*t1**3
   + (z/4 + y*theta/4)*t1**4
   + (z*theta/5)*t1**5
   - w*sigma1/(sigma2+1)*t1**(sigma2+1)
   - x*sigma1/(sigma2+2)*t1**(sigma2+2)
   - y*sigma1/(sigma2+3)*t1**(sigma2+3)
   - z*sigma1/(sigma2+4)*t1**(sigma2+4)
)

# B(t1)
B_expr = (
     w/2*t1**2
   + (x/3 + w*theta/6)*t1**3
   + (y/4 + x*theta/8)*t1**4
   + (z/5 + y*theta/10)*t1**5
   + (z*theta/12)*t1**6
   - w*sigma1*sigma2/((sigma2+1)*(sigma2+2))*t1**(sigma2+2)
   - x*sigma1*sigma2/((sigma2+1)*(sigma2+3))*t1**(sigma2+3)
   - y*sigma1*sigma2/((sigma2+1)*(sigma2+4))*t1**(sigma2+4)
   - z*sigma1*sigma2/((sigma2+1)*(sigma2+5))*t1**(sigma2+5)
)

# Shortage term
Shortage_expr = sp.integrate(R * sp.exp(-delta*(T - t)), (t, t1, T))

# L(t1)
L_expr = (
     w*sigma1/(sigma2+1)*t1**(sigma2+1)
   + x*sigma1/(sigma2+2)*t1**(sigma2+2)
   + y*sigma1/(sigma2+3)*t1**(sigma2+3)
   + z*sigma1/(sigma2+4)*t1**(sigma2+4)
```

```
)

# Total cost
TC_expr = (
      A0
    + py * Q_expr
    + hy * B_expr
    + zy * theta * B_expr
    + wy * L_expr
    + gamma * B_expr
    + rho * B_expr
    + omega * Q_expr
    + ys * Shortage_expr
)

# Derivative
dTC_dt1 = sp.diff(TC_expr, t1)

# Numeric solve
t_guess = 0.25
t_star = float(sp.nsolve(dTC_dt1, t1, t_guess))

# Compute values
Q_star = float(Q_expr.subs(t1, t_star))
TC_star = float(TC_expr.subs(t1, t_star))

print("t* =", t_star)
print("Q* =", Q_star)
print("TC* =", TC_star)
```

OUTPUT

```
t* = 0.2544585513141862
Q* = 92.12883047933177
TC* = 75586426.73736925
```